Some Results in Ordinal Ramsey Theory

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Thursday, 3$^{\text{rd}}$ February 2022

Abstract

The Ramsey Theory of noninitial ordinals is substantially less explored than the Ramsey Theory of cardinals. Certain cases were reduced to Ramsey statements involving finite oriented graphs by (Erdős and Rado) and Baumgartner. Deepak Rajendraprasad, Ferdinand Ihringer and I managed to determine two of the hitherto unknown values thus answering a question by J. Larson and Mitchell by both improving on their quadratic upper bound and finding concrete examples yielding lower bounds. We also established an upper bound of the asymptotically correct order of magnitude $\dfrac{n^2}{\log(n)}$.

Recently I worked on improving the constant in this bound and started investigating the differences between consecutive Ramsey numbers of the kind in question. I am going to report on my progress.

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